Engineering Notes

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Emission Spectra from a Stream of Cooling Particles

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Nomenclature

λ = wavelength, cm
T = temperature, °K
$W_{BB}(\lambda, T)$ = spectral radiance $= 1.1905 \times 10^{-12}/\lambda^5[\exp(1.4384/\lambda T) - 1]$ W/cm^2-steradian-cm
$\varepsilon(\lambda)$ = spectral emissivity
C_p = molar heat capacity at constant pressure, cal/mole-°K
$\dot{W}$ = mass flow rate, lb/sec
M = molecular weight, g/mole
m = mass of particle, g
A_p = surface area of a particle, cm^2
E_T = total radiant power from a particle
r = particle radius, cm
ρ = particle density, g/cm^3
ε_p = particle emissivity $= Q_a = \sigma_a/\pi r^2$
σ_a = particle cross section for absorption, cm^2
m = complex index of refraction $= n - jk$
n = real part of index of refraction
k = imaginary part of index of refraction = extinction coefficient

Introduction

THE prediction of emission from luminous flames has several applications in combustion and rocket technology. A frequent example is the calculation of the radiant heating of regions adjacent to the exhaust of a rocket nozzle. A significant part of the complete aerothermochemical analysis is the determination of the spectral radiance of particulate matter in the flame or plume.[1,2] An essential problem in the analysis of particulate radiation is the determination of particle or cloud emissivity. Hence, quite a number of experimental and/or theoretical studies have been undertaken on the emissive or absorptive properties of particles of carbon[3-6] and oxides.[7-12] In this Note an analytical technique is presented for the calculation of the spectral radiant emission from hot particles ejected into a vacuum in which total radiant energy is determined from thermodynamic considerations and modified by a spectral emission efficiency to obtain the spectral radiant emission. An example is given for carbon particles for which emissivities are calculated from an appropriate approximation to Mie theory. A spectral emission curve is obtained between 1- and 28-μm wavelengths for cooling from a maximum of 2500°K to 300°K and a persistence time and length are calculated.

Method of Analysis

Interest is confined here to an idealized system consisting of uniformly sized particles being ejected in a constant-velocity stream from a heated container into a vacuum. Such a system may be taken as an approximation to particle emission in rocket exhausts at altitudes above 100 km where collision rates are low, convective cooling and aerodynamic reheating of particles are negligible as compared with radiative heat loss, and the particle cloud is optically thin. Spectral emissivity is used in an efficiency factor to modify the net enthalpy change between two temperatures for a given mass flow-rate of particles. Thus, the correct total emission is ensured and the analysis is directed to finding the best estimate of the spectral radiant energy distribution.

Radiant power formula

For a particle radiating at temperature, T, the fraction of energy emitted in the wavelength range between λ_1 and λ_2 can be designated by

$$\phi_{12}(T) = \int_{\lambda_1}^{\lambda_2} \varepsilon_p(\lambda, T) W_{BB}(\lambda, T)\, d\lambda \Bigg/ \int_0^{\infty} \varepsilon_p(\lambda, T) W_{BB}(\lambda, T)\, d\lambda \qquad (1)$$

If, as we assume, the particle is cooling by radiation only, the energy radiated at T to cause a drop in temperature, dT, is just the change in enthalpy

$$dH = C_p(T)\, dT \text{ cal/mole} \qquad (2)$$

and the change in enthalpy due to radiation in the designated band is

$$dH_{12} = \phi_{12}(T) C_p(T)\, dT \qquad (3)$$

For a steady flow of particles cooling from a chamber exhaust temperature, T_0, to a final equilibrium temperature, T_1, the radiant power emitted in the band is given, for the optically thin assumption, by

$$E_{12} = (151 \dot{W}/M) \int_{T_1}^{T_0} \phi_{12}(T) C_p(T)\, dT \; W/\text{steradian} \qquad (4)$$

The 151 is the necessary conversion factor $(453.6 \times 4.186/4\pi)$ between (lb-mole-cal/gm-mole-sec) and (w/steradian). T_0 represents the initial temperature of the particles. For a rocket nozzle exhausting at high altitudes this temperature normally becomes determined near the exit plane where collisions between gas ion species and particles are so slow that convective heating becomes unimportant. However, in this present case T_0 is considered as a given parameter. T_1, the final equilibrium particle temperature, normally must be calculated for the particle case being considered. Thus the altitude is important, since the degree of long wavelength Earth radiation which can reheat the particle becomes less as one goes further away from the Earth. Also, the particle emissivity is important. For many problems of interest final particle temperatures of about 200–300°K are expected.

An interesting observation that can be confirmed with Eq. (4) is that, for equal temperature drops, many materials have about the same total radiant power available (for $\phi_{12} = 1$). This is shown in Table 1 for three materials of interest. The spectral distribution of this power will vary with the species.

Table 1 Total radiated power (W/steradian) for various materials and temperature changes (a flow rate of 0.001 lb/sec is assumed)

$T_0 - T_1$ (K)	C	MgO	Al_2O_3
2500–300	143	102	...
2320–300	...	...	99
2000–300	106	77	74
1500–300	70	53	50

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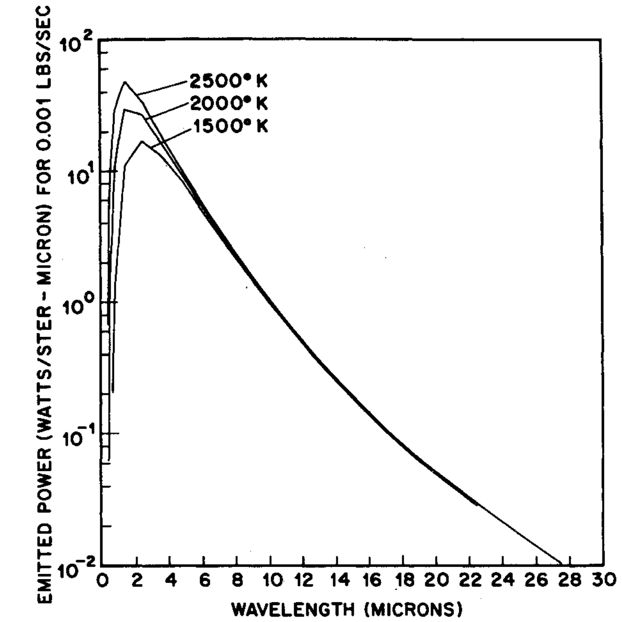

Fig. 1 Predicted spectral emission of a cooling stream of 0.02-μm-diam carbon particles for three initial temperatures (flow rate 0.001 lb/sec; final temperature 300°K; initial temperatures as shown).

Persistence time‡

The distance over which a group of cooling particles is able to radiate is a useful quantity. It equals the product of the particle velocity and the persistence time, t_p, necessary for the particle to cool (by radiation) between the specified temperatures. From

$$(mC_p/M)\,dT/dt = A_p E_T \tag{5}$$

it can be seen that

$$t_p = \frac{\rho r}{3\pi M}\int_{T_1}^{T_0}\left[C_p(T)\,dT\bigg/\int_0^\infty \varepsilon_p(\lambda, T)W_{BB}(\lambda, T)\,d\lambda\right] \tag{6}$$

The π comes from integrating W_{BB} over a hemisphere. This equation gives the expected result that small particles with low specific heat and high emissivity cool most rapidly.

Particle emissivity approximation for carbon

Exact theory would require the use of the Mie infinite series of Bessel function terms[13] to determine emissivity from n and k. However, for values of $x = 2\pi r/\lambda$ less than about 0.6, there is a good approximation available for Q_a,[14]

$$Q_a = -I_m\left[4x\frac{m^2-1}{m^2+2} + \frac{4x^3}{15}\left(\frac{m^2-1}{m^2+2}\right)^2 \frac{m^4+27m^2+38}{2m^2+3}\right] \tag{7}$$

This is good over the whole wavelength range of IR radiation for carbon particles which have a diameter of about 0.02μm and, in fact, the second term in the expression is unnecessary. Taking the imaginary part of the first term, one obtains

$$Q_a = (48\pi r/\lambda)\cdot nk/[(n^2-k^2+2)^2+4n^2k^2] \tag{8}$$

For this linear dependence on r, ϕ_{12} is independent of particle size and size distribution. Also, it can be seen from Eq. (6) that the persistence time is independent of particle size. A computer program was written to calculate Q_a for carbon from Eq. (8) using the temperature-dependent expressions for n and k for fitting the experimental data which were obtained by Boynton et al.[6] Also, the modifications which they observed at elevated temperatures were incorporated by using the following empirical formula:

$$Q = Q_a\left(\frac{\lambda^5+1.61}{1.61}\right)^{3.04\times10^{-4}(T-1675)} \quad \text{for} \quad T \geqq 1675 \tag{9}$$

‡ The phrase "persistence" implies that even if the flow of particles is suddenly stopped, the radiation will persist for a time designated as t_p.

Calculations

Optical property data

The temperature range considered was for cooling from a maximum of 2500°K down to 300°K. In order to make a valid numerical integration approximation to the infinite wavelength range, it is necessary to consider emissivity values at wavelengths beyond the range where blackbody radiation at the above temperatures is significant. We chose to integrate between 0.2 and 100 μm. The values for n and k for carbon were obtained using the formulas mentioned in the previous discussion.

Emission spectra

The predicted spectral emission of a flowing stream of carbon particles is shown in the figure. Three cases are shown for cooling to 300°K from 1500°K, 2000°K, and 2500°K. Other assumed conditions were a flow rate of 0.001 lb/sec and a particle diameter of 0.02 μm. The spectra are shown over the range of significant radiation between about 1- to 28-μm wavelength. Carbon particles, with a relatively high emissivity over the whole wavelength range, give a spectral distribution characteristic of the sum of gray-body emission over the temperature range. Thus, it peaks at a low wavelength, corresponding to the highest particle temperature, and falls off at higher wavelengths as one would expect from the Wien's Law relationship. The persistence time, calculated from Eq. (6), and length for a velocity of 2500 fps and for cooling from 2500°K to 300°K are 0.27 sec and 675 ft, respectively.

Clearly, the technique described here can be applied to the calculation of the emission spectra for other particle species. Preliminary results have been obtained for several oxides, but these will not be reported until more accurate and dependable values for the optical properties have been determined.

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